MODELIZACIÓN DE LA VOLATILIDAD CONDICIONADA EN EL ÍNDICE BURSÁTIL ESPAÑOL IBEX-35 EMPLEANDO DATOS DE ALTA FRECUENCIA. UNA COMPARACIÓN ENTRE EL MODELO EGARCH Y LA RED NEURONAL BACKPROPAGATION

MODELLING CONDITIONAL VOLATILITY IN THE SPANISH IBEX-35 STOCK INDEX USING HIGH FREQUENCY DATA. A COMPARISON OF THE EGARCH MODEL AND THE BACKPROPAGATION NEURAL NETWORK

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Resumen:

El análisis de la volatilidad condicionada es un paso necesario para poder valorar de forma precisa el riesgo inherente a activos financieros tales como acciones, bonos, índices, derivados etc. Una buena predicción de la volatilidad es necesaria para la correcta diversificación de carteras de inversión, para calcular el valor de las opciones, del VaR. etc. Por este motivo es necesario generar modelos capaces de predecir la volatilidad de los activos financieros. En la actualidad, los modelos que se emplean más habitualmente son los modelos econométricos de la familia GARCH. En este artículo se analiza la volatilidad condicionada del índice bursátil español IBEX-35 con datos de alta frecuencia mediante el modelo ARMA-EGARCH, que permite captar las asimetrías en la volatilidad del índice. Seguidamente se emplea la red neuronal BPN sobre la misma muestra de datos y se comparan los resultados obtenidos. La comparación se realiza utilizando las mismas variables en ambos modelos para poder obtener una comparación más equilibrada y justa. Los resultados muestran que la red neuronal es una buena alternativa a los modelos econométricos de la familia GARCH. De hecho, en el análisis realizado, la red neuronal backpropagation bate en repetidas ocasiones al modelo ARMA-EGARCH independientemente de la frecuencia de los datos y de la forma en que se mide el error de predicción.

Palabras clave: volatilidad, EGARCH, red neuronal backpropagation, índice bursátil IBEX-35, alta frecuencia

Abstract:

The analysis of conditional volatility is a necessary step towards the accurate valuation of the risk inherent in financial assets such as stocks, bonds, indices and derivatives among others. A good volatility prediction is required to diversify portfolios, value financial options, calculate VaR, etc. Therefore, it is necessary to generate models that are capable to predict financial assets' volatility. At present, the most widely employed models are those belonging to the GARCH family. In this paper the conditional volatility of the Spanish IBEX-35 stock index with high frequency data is analyzed by means of an ARMA-EGARCH model, as this model can capture the asymmetries in the index volatility. Next we apply the backpropagation neural network to the same end and compare the results obtained. The comparison is made using the same variables in both models in order to obtain a more balanced and fair comparison. The results show that the neural network is a good alternative to the traditional GARCH family models. In fact, in the analysis, the backpropagation neural network repeatedly beats the ARMA-EGARCH model regardless the frequency of the data and the error measurement type employed.

Keywords: Volatility, EGARCH, neural network, backpropagation, IBEX-35 stock index, high frequency

1. INTRODUCCIÓN

The study of present and future volatility of financial assets is a topic which has devoted the attention of researchers and practitioners along the past decades (Oliver, 2014). The reasons for this interest are manifold. Some authors state that the volatility in the financial markets can have an impact on the real economy (Bernanke, 1983; Gertler, 1988). Furthermore, volatility plays a key role in portfolios' diversification given that the risk of the portfolio is determined by the evolution and the intensity of the volatility. For that reason, an increase in volatility may affect the assets selected in a portfolio (Solnik, 1974; Grauer *et al.*,1987). In addition to this, volatility is a key input for the calculation of the Value at Risk (VaR) and for the Black and Scholes (1973) and Merton (1973) options' valuation model.

The aim of this paper is to compare the ability of the ARMA-EGARCH model and the neural network backpropagation to forecast the conditional variance of the Spanish Ibex-35 stock index using high frequency data. Most of the previous comparative studies in the literature do not use the same inputs in the ARMA-EGARCH model and the neural network backpropagation, as more inputs are introduced in the neural network (Hossain *et al*., 2010; Maknickiene and Maknickas, 2013). In order to undertake a more balanced comparison we have used the same inputs in both methodologies. The results obtained confirm that the neural network can be a good alternative to the traditional models from the GARCH family to predict volatility in stock indices with high frequency data. In fact, the predictions by the neural network beat those by the econometric model. This result is shared by other studies that focus on stock index conditional volatility prediction using daily data (Lahmiri, 2012; Hang and Wang, 2010; Vejendla and Enke, 2013).

The remainder of the paper is structured as follows. Section 2 describes the econometric model ARMA-EGARCH and the neural network backpropagation. Section 3 applies both methodologies to the analysis of the conditional volatility of the Spanish IBEX-35 stock index using high frequency data. Finally, section 4 concludes.

2. DESCRIPTION OF THE METHODOLOGY

2.1. ARMA-EGARCH MODEL

The models of the GARCH family have been widely applied to model and forecast volatility in financial time series. The Autorregresive Conditional Heteroskedaticity (ARCH) model by Engle (1982) and the general model (GARCH) by Bollerslev (1986) were developed to relax the assumption of constant variance through time. That is, they introduce a conditional variance that depends on the available information and that varies along time in function of the past residuals, keeping the inconditional variance constant.

Later, Nelson (1990) proposed the EGARCH (Exponential GARCH) model to capture the asymmetric effect on the volatility of financial assets of good and bad news. Good news increase volatility less than bad news. Compared to the GARCH model, the EGARCH model needs no restrictions to be imposed for its estimation. The EGARCH model is described as follows:

$$\log(h_t) = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \left[\gamma \frac{\varepsilon_{t-j}}{\sqrt{h_{t-j}}} + \left(\frac{|\varepsilon_{t-j}|}{\sqrt{h_{t-j}}} - (2/\pi)^{1/2} \right) \right]$$

Where α_0, α_i, β_j, γ are coefficients to be estimated, h_{t-j} is the conditional variance and ε are the lagged disturbances.

The third term in the equation represents the expected value of $\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}$ suppossing a Normal distribution (0,1), being $\mathrm{E}\left[\left|\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}\right|\right] = \left(\frac{2}{\pi}\right)^{1/2}$.

In our research, the EGARCH model has been prefered to other asymmetric models because in the EGARCH model there is no positivety restriction on the parameters. This restriction does hold in other models like the Asymmetric GARCH or the model by Glosten, Jagannathan and Runkle (GJR). Compared to the Threshold GARCH (TGARCH) model, the EGARCH model has always a positive variance The TGARCH model uses the standard deviation, which may have negative values that can be difficult to interpret (Haoliu *et al.*, 2009; Liu and Hung, 2010).

2.2. BACKPROPAGATION NEURAL NETWORK

The prediction ability of the ARMA-EGARCH model presented above will be compared with the prediction power of an artificial neural network.

Artificial neural networks can be defined as a series of mathematical algorithms which aim to find out non-linear relationships among a determined dataset. They are based on the behaviour of biological neurons. One of the most widely neural networks used to estimate returns and conditional volatility is the backpropagation neural network proposed by Rumelhart *et al*. (1986) which has been used in studies by (Ariyo *et al.,*2014; Sim *et al.*,2014; Pawar *et al.*, 2014).This network applies a delta rule-based supervised learning. The learning algorithm of the generalized delta rule is expressed as follows:

$$\Delta w_{ij}(t+1) = \alpha\delta_{pj}y_{pi} + \beta\Delta w_{ij}(t) \quad (2)$$

where:

α – is the learning factor with values between 0 and 1. This parameter determines the learning speed of the neuron and its value will remain constant.

y_{pi} – is the output value of neuron i under the learning pattern p.

δ_{pj} – is the value of delta or the difference between the desired output value and the value actually obtained by the neural network.

β – is a constant that determines the effect in t+1 of the change in the weights in time t. With this constant a better convergence is achieved with less iterations.

The implementation of the backpropagation algorithm or generalized delta rule requires the use of neurons with a continuous and differentiable activation function. This function is normally sigmoid or tan-sigmoid, but linear functions can be employed as well.

3. RESULTS

The models introduced in the previous section have been applied to forecast the conditional volatility of the Spanish IBEX-35 stock index using high frequency data. The frequencies used are 5, 10, 15, 30 and 60 minutes.

The sample analyzed ranges from January 2000 until December 2010. From this sample, five estimation and prediction subsamples have been generated. A period of five years is always used for the estimation of the prediction model and the next year is used to make the prediction (Table 3.1).

Table 3.1. Estimation subsamples and predicted year

	Estimation		Prediction
Subsample	Initial year	Ending year	Year
1	2000	2005	2006
2	2001	2006	2007
3	2002	2007	2008
4	2003	2008	2009
5	2004	2009	2010

The selected sample is wide enough to cover a period including different market trends like bull market, bear market and lateral market (Figure 3.1).

Figure 3.1. Ibex-35 stock index daily data. Period: 2000-2010

Source: Visualchart

In order to compare the predictive power of the econometric model and the neural network, the errors made by each model for every sample are calculated. Four different types of prediction error measures are used: Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE), Mean Percentage Error (MPE) and Root Mean Square Error (RMSE), which are calculated as follows:

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|\hat{y}_i - y_i|$$

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{\hat{y}_i - y_i}{y_i}\right|$$

$$MPE = \frac{1}{n}\sum_{i=1}^{n}\frac{\hat{y}_i - y_i}{y_i}$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\hat{y}_i - y_i)^2}$$

Where y is the observed volatility and $\hat{y}_i$ is the predicted volatility.

In order to select the most appropriate econometric model, several analyses must be undertaken previously.

The first one is to determine whether all the series in the subsamples are stationary. To this end the augmented unit-root test by Dickey-Fuller (DFA) and the Phillips-Perron test are used. The tests are applied 1) without constant and without tendency; 2) with constant; and 3) with constant and with tendency. In order to calculate the lags to conduct the tests, the Schwartz criterion is employed. The tests are calculated with a level of significance of 1%, 5% y 10%. The non-existence of unitary roots confirms the stationarity of the series regardless the test employed. All the data series are stationary as the statistical values reject the null hypothesis of the existence of unitary roots both using the DFA test as the Phillips-Perron test.

Once the stationarity of the series has been verified, the next step consists on the analysis of the autocorrelation of the residuals. To this end a regression with different lagged values, from one to five, is calculated for each of the series. The test employed is the Breusch-Godfrey contrast. For all series a critical value is obtained that confirms the existence of autocorrelation of the residuals.

Then, different ARMA models with different lags are estimated and the best one is chosen using the Schwartz criterion. The heteroscedasticity of the residuals is analyzed by the ARCH test. In all cases the null hypothesis is rejected and the existence of heteroscedasticity of the residuals is confirmed. So it is appropriate to use an econometric model from the GARCH family.

An ARMA-EGARCH model is estimated for each of the datasets (5, 10, 15, 30 and 60 minutes) where different lags have been selected using the Schwartz criterion (Table 3.2).

Table 3.2. Selected EGARCH models according to the Schwartz criterion

INDEX	5min	10min	15min	30min	60min
IBEX-35	ARMA(1,1)-EGARCH(2,2)	ARMA(1,2)-EGARCH(2,2)	ARMA(1,1)-EGARCH(2,2)	MA(1)-EGARCH(2,2)	ARMA(1,1)-EGARCH(1,1)

Table 3.3 summarizes the prediction errors made by the models in each of the 5 subsamples and for the different data frequencies. No correlation has been confirmed between the volatility in each subsample and the prediction error made by the model.

Table 3.3. Prediction errors by the EGARCH model using IBEX-35 stock index high frequency data

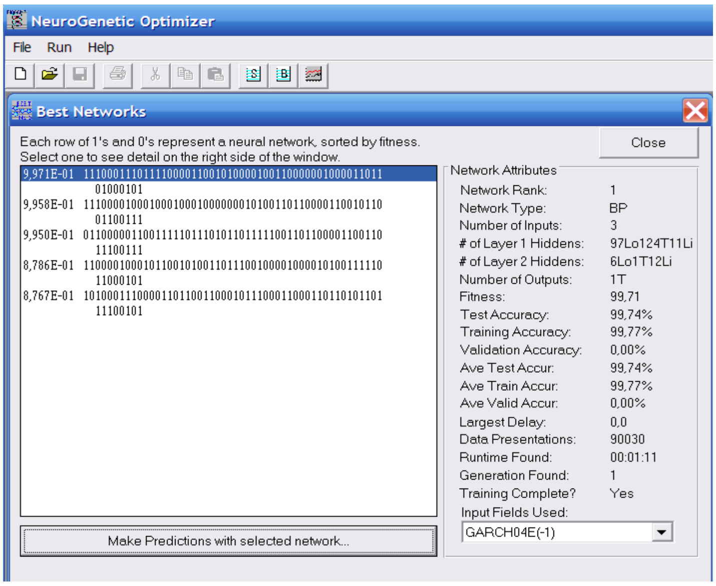
		MAPE	MAE	MPE	RMSE
IBEX (5 min)	V1	0.26248416	9.0361E-06	0.20301436	0.00300601
	V2	0.31974141	4.0943E-06	0.31059916	0.00202343
	V3	0.27169359	0.00021669	0.21694075	0.01472031
	V4	25.7085722	0.00011302	-25.61777	0.0106311
	V5	0.210992	5.6018E-	0.0599885	0.0074845

		98	05	8	3
IBEX (10min)	V1	0.33197646	9.3775E-07	0.32012453	0.00096838
	V2	0.70192234	2.3362E-06	0.01836914	0.00152848
	V3	0.39127714	8.8449E-06	0.18583482	0.00297404
	V4	0.63759945	2.3927E-06	-0.6032699	0.00154682
	V5	0.43904343	4.8223E-06	0.10976611	0.00219597
IBEX (15min)	V1	0.81701219	2.2337E-06	0.15434504	0.00149457
	V2	0.53227474	2.5974E-06	0.36421663	0.00161163
	V3	0.62686993	1.0798E-05	0.54322686	0.00328599
	V4	0.1819591	1.3007E-06	-0.12425093	0.00114047
	V5	0.34080232	3.7072E-06	0.17380106	0.00192541
IBEX (30 min)	V1	0.61524793	1.7293E-06	0.09932522	0.00131504
	V2	0.15390362	9.3501E-07	0.11663814	0.00096696
	V3	0.21428101	6.7583E-06	0.20897526	0.00259967
	V4	0.42205606	5.429E-06	-0.23938849	0.00233003
	V5	0.31211814	4.4173E-06	-0.17310016	0.00210174
IBEX (60 min)	V1	0.01255168	7.6402E-08	0.00892704	0.00027641
	V2	0.0254638	3.0464E-07	0.00883194	0.00055194

	V3	0.04831693	3.2911E-06	0.04465462	0.00181413
	V4	0.02810043	6.9662E-07	-0.00081787	0.00083464
	V5	0.00975933	3.5436E-07	0.00432106	0.00059529

Next, the prediction of conditional volatility is calculated by means of the backpropagation neural network. To this end, the same inputs have been used as in the econometric models from the ARMA-EGARCH family above. The neural network has been trained for each of the five estimation and prediction subsamples. The computation time required to obtain a neural network able to beat the econometric model has been relatively low. So, for example, figure 3.2 shows a computation time of one minute and eleven seconds for the subsample 4 for a data frequency of 15 minutes. The network obtained beats the prediction errors regarding the analog econometric model.

Figure 3.2. Computation time of the neural network for the IBEX-35 stock index using 15 minutes frequency data for subsample 4: 1 minute and 11 seconds.



Source: The authors

In general, the transfer functions in the neurons of the output layer in all the networks obtained are linear functions. Regarding the hidden layers, the transfer functions are balanced: 85 neurons have linear transfer functions, 85 have a logarithmic transfer function and 85 a tangent transfer function. This balanced structure has improved the predictions obtained by the econometric model.

Table 3.4 summarizes the prediction errors made for each subsample and for each data frequency applying the backpropagation neural network. As was already the case when the econometric model was applied, prediction errors are independent of the higher or lower volatility experienced in the subsamples.

Table 3.4. Prediction errors by the neural network using IBEX-35 stock index high frequency data

		MAPE	MAE	MPE	RMSE
IBEX (5 min)	V1	0.16103917	3.0668E-06	-0.02933468	0.00175123
	V2	0.17242818	2.8873E-06	0.03584239	0.0016992
	V3	0.27342705	1.7364E-05	0.07745063	0.00416699
	V4	0.23439164	3.3767E-07	-0.14522657	0.00058109
	V5	0.63494582	4.2718E-06	0.10811331	0.00206683
IBEX (10 min)	V1	0.13468692	2.5518E-07	-0.02309185	0.00050516
	V2	0.14793804	6.4137E-07	0.00019941	0.00080086
	V3	0.17342324	4.2905E-06	0.07763179	0.00207136
	V4	0.18696436	1.2058E-06	-0.00231229	0.00109808
	V5	0.03275266	7.8515E-07	-0.00164589	0.00088609
IBEX (15	V1	0.08055867	2.532E-07	0.02132416	0.00050319

min)	V2	0.08003144	4.7326E-07	0.01927263	0.00068794
	V3	0.14622448	3.2668E-06	0.13318769	0.00180743
	V4	0.07144154	3.2629E-07	-0.04515404	0.00057122
	V5	0.09918139	1.0461E-06	0.01012666	0.00102281
IBEX (30 min)	V1	0.0351413	1.3121E-07	0.02362043	0.00036223
	V2	0.03362204	1.9099E-07	0.01231117	0.00043702
	V3	0.12621481	6.7877E-06	0.10704967	0.00260533
	V4	0.04002451	5.4477E-07	0.02426932	0.00073808
	V5	0.05703196	8.8434E-07	-0.0011203	0.0009404
IBEX (60 min)	V1	0.0099651	5.6151E-08	-0.00088317	0.00023696
	V2	0.01206509	1.3376E-07	0.00042134	0.00036573
	V3	0.01792584	1.1761E-06	0.00168591	0.00108449
	V4	0.01834084	5.3136E-07	0.0013001	0.00072895
	V5	0.0217989	7.5782E-07	0.00033423	0.00087053

When the prediction errors made by the ARMA-EGARCH model and the backpropagation neural network are compared, it can be noticed that the neural network improves the predictions of the econometric model for most of the subsamples. Table 5shows the error reduction obtained by the neural networks compared to the ARMA-EGARCH models in percentage terms.

Table 3.5. Reduction of the prediction error in percentage. Neural network backpropagation vs. ARMA-EGARCH model. IBEX-35 stock index high frequency data

		MAPE	MAE	MPE	RMSE
IBEX (5 min)	V1	-38.6%	-66.1%	-85.6%	-41.7%
	V2	-46.1%	-29.5%	-88.5%	-16.0%
	V3	0.6%	-92.0%	-64.3%	-71.7%
	V4	-99.1%	-99.7%	-99.4%	-94.5%
	V5	200.9%	-92.4%	80.2%	-72.4%
IBEX (10 min)	V1	-59.4%	-72.8%	-92.8%	-47.8%
	V2	-78.9%	-72.5%	-98.9%	-47.6%
	V3	-55.7%	-51.5%	-58.2%	-30.4%
	V4	-70.7%	-49.6%	-99.6%	-29.0%
	V5	-92.5%	-83.7%	-98.5%	-59.6%
IBEX (15 min)	V1	-90.1%	-88.7%	-86.2%	-66.3%
	V2	-85.0%	-81.8%	-94.7%	-57.3%
	V3	-76.7%	-69.7%	-75.5%	-45.0%
	V4	-60.7%	-74.9%	-63.7%	-49.9%
	V5	-70.9%	-71.8%	-94.2%	-46.9%
IBEX (30 min)	V1	-94.3%	-92.4%	-76.2%	-72.5%
	V2	-78.2%	-79.6%	-89.4%	-54.8%
	V3	-41.1%	0.4%	-48.8%	0.2%
	V4	-90.5%	-90.0%	-89.9%	-68.3%
	V5	-81.7%	-80.0%	-99.4%	-55.3%
IBEX (60 min)	V1	-20.6%	-26.5%	-90.1%	-14.3%
	V2	-52.6%	-56.1%	-95.2%	-33.7%
	V3	-62.9%	-64.3%	-96.2%	-40.2%
	V4	-34.7%	-23.7%	59.0%	-12.7%
	V5	123.4%	113.9%	-92.3%	46.2%

Table 3.5 shows that the neural network has clearly improved the predictions of the econometric model in 80% of the estimated subsamples. Out of the 25 prediction subsamples, only in 5 of them has the econometric model obtained a smaller prediction error than the neural network according to at least one of the four error measures employed (table 3.6). Out of these five cases, in three of them the econometric model has beaten the neural network regarding just one error

measurement type. In another case, the econometric model beats the neural network regarding two error measurement types and in the last case regarding three error types.

Table 3.6. Neural network vs. Econometric model

		MAPE	MAE	MPE	RMSE
IBEX 5 min	V3	0	1	1	1
IBEX 5 min	V5	0	1	0	1
IBEX 30 min	V2	1	1	0	1
IBEX 60 min	V4	1	1	0	1
IBEX 60 min	V5	0	0	1	0

1: neural network beats econometric model; 0: econometric model beats neural network

It can be concluded that the predictions of the neural network clearly beat those of the econometric model. This result agrees with the findings of previous researches (Sun, 2007; Sun *et al.* 2010) that compare the backpropagation neural network and the ARMA-GARCH model using intraday data from the German DAX-30 index to calculate the VaR. Similarly, Trebaol (2010) studies the intraday volatility in the French CAC-40 index, obtaining that the neural network forecast beats the forecast by the econometric model. As in the present paper, the mentioned studies apply intraday index data to compare the prediction capacity of neural networks versus econometric models. Nevertheless, our research differs from the cited studies because they introduce more inputs in the neural network than in the econometric model. Therefore the comparison of both methodologies is not really balanced as they do not use the same amount of information.

4. CONCLUSIONS

In this paper we compare the ability of the ARMA-EGARCH model and the neural network backpropagation to forecast the conditional variance of the Spanish Ibex-35 stock index using high frequency data. Other than in previous studies devoted to the same topic, in our research we undertake the comparison in a balanced way introducing the same variables in both models. This is not the case in other studies in the literature which use more and/or different variables in the neural network than the econometric model. Furthermore, most of these works analyze the conditional volatility on daily data, not with high frequency data as in this paper. Our study concludes that the predictions of the neural network clearly beat those of the econometric model showing that the neural network is a good alternative to the traditional GARCH family models.

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