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An estimation of cost-based market liquidity from daily high, low and close prices

Una estimación de la liquidez de mercado basada en los costes a partir de los precios máximo, mínimo y de cierre

Jawad Saleemi [1,2]

[1]*Business School, University of Lahore. Lahore, Pakistan. Email: j.saleemi@yahoo.com*
[2]*Economics and Social Sciences Department, Universitat Politècnica De Valéncia. Valencia, Spain. Email: Jasa1@doctor.upv.es*

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Abstract

In the literature of asset pricing, this paper introduces a new method to estimate the cost-based market liquidity (CBML), that is, the bid-ask spread. The proposed model of spread proxy positively correlates with the examined low-frequency spread proxies for a larger dataset. The introduced approach provides potential implications in important aspects. Unlike in the Roll bid-ask spread model and the CHL bid-ask estimator, the CBML model consistently estimates market liquidity and trading cost for the entire dataset. Additionally, the CBML estimator steadily measures positive spreads, unlike in the CS bid-ask spread model. The construction of the proposed approach is not computationally intensive and can be considered for distinct studies at both market and firm levels.

Keywords: Market Microstructure; Asset Pricing; Bid-Ask Spread; Market Liquidity; Trading Cost

Resumen

Este documento presenta un nuevo método, en la literatura sobre fijación de precios de activos, para estimar la liquidez del mercado basada en el coste (CBML), es decir, el diferencial entre oferta y demanda. El modelo propuesto con un proxy del diferencial (spread) se correlaciona positivamente con los proxy del diferencial de baja frecuencia examinados para un conjunto de datos más grande. El enfoque introducido proporciona potenciales implicaciones en aspectos importantes. A diferencia del modelo de diferencial de oferta y demanda y el estimador CHL, el modelo CBML estima constantemente la liquidez del mercado y el costo comercial para todo el conjunto de datos. Además, el estimador CBML mide constantemente los diferenciales positivos, a diferencia del modelo de diferencial de oferta y demanda CS. La construcción del enfoque propuesto es asumible computacionalmente y puede considerarse para estudios distintos tanto a nivel de mercado como de empresa.

Keywords: Microestructura del mercado; Fijación de precios de activos; Diferencial de oferta y demanda; Liquidez del mercado; Coste de negociación

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Corresponding author
Jawad Saleemi

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1 Introduction

This paper provides new insights of estimating the cost-based market liquidity in financial markets. Starting with the Roll bid-ask spread model, a spread has been widely considered for the estimation of cost-based market liquidity at the time of trade and future trading sessions. The bid-ask spread is a useful indicator of executing cost faced by investors, and thus, a proxy for market liquidity (Corwin y Schultz, 2012). In the literature of asset pricing, distinct models have been proposed to framework the bid-ask spread. However, some shortcomings have been identified in various bid-ask spread models (Goyenko, Holden, y Trzcinka, 2009).

In this paper, the CBML model is another version of low-frequency spread proxies. The rationale of the introduced estimator is based on simple foundation, that is, a spread can be determined by various components, including information asymmetry cost, immediacy cost, and order processing cost. The proposed approach considers a wider set of information consisted in daily high, low, and close prices, while constructing the possible presence of an informed trader. Additionally, the CBML model looks at pre-and-post-trade prices by a logic that the liquidity providers would be compensated against the provision of riskier liquidity and administration expenses. Most importantly, the CBML estimator is not computationally intensive and provides comprehensive implications for both academics and those who participate in trading.

To construct the CBML estimator, the model considers distinct theoretical assumptions: (a) high prices and low prices are always initiated by buyers and sellers, respectively (Corwin y Schultz, 2012); (b) an informed trader, either from buyer-side or seller-side, is always present with equal probability in the market (Glosten y Milgrom, 1985); (c) a transaction discloses inventory holding cost that liquidity providers demand against the provision of price fluctuations (Amihud y Mendelson, 1980); and (d) liquidity providers would also be compensated for the order processing cost at the time of trade (Roll, 1984). Based on these theoretical assumptions, the CBML model is constructed in the following analytical steps:

$$S_t = \left(\frac{H_{t-1} - L_{t-1}}{C_{t-1}} \right) - \left(\frac{v_t^H - v_t^L}{C_t} \right) \tag{1}$$

Where, S_t is the bid-ask spread, and derived by the difference between the ratio of an asset's range to its close price on day $t-1$ and the ratio of an informed asset's range to its close price on day t. S_t reflects the ease and cost of trading on day t. In the proposed model, H_{t-1} and L_{t-1} denoted to the highest price asked by a seller and the lowest price that a buyer was willing to pay against the asset on day $t-1$, respectively. C_{t-1} is the closed price of an asset on day $t-1$.

For the following trading session, it is assumed that an informed trader would impact prices. Assuming risk neutrality, the asset is valued in the following trading session at:

$$\eta_t = \frac{H_t + L_t}{2} \tag{2}$$

Where, η_t is the mean of high and low prices on day t. Understanding the equal probability of an informed trader, the expected highest value for which a seller would sell the security is assumed conditional on a trade at:

$$v_t^H = H_t \pi + \eta_t \pi \tag{3}$$

Where, the expected lowest price that a buyer would pay against the asset is assumed conditional on a trade at:

$$v_t^L = L_t\pi + \eta_t\pi \tag{4}$$

The model further encounters the relationship of an informed asset's range to its closed price, C_t. The CBML model assumes that a ratio of asset's range to close price would be greater in case of higher probability of trading with an informed trader on day t. This model looks at past prices by a logic that providers of liquidity would be compensated in the following trading session against the price fluctuations and administration expenses. This implies, the CBML estimator reflects cost-based market liquidity for two-consecutive single days. Within this framework, the model assumes the volatility factor by computing the variance of spread and then taking the square root of it.

$$CBML_t = \sqrt{S_t^2} \tag{5}$$

$CBML_t$ reflects market liquidity, trading cost, and volatility for the financial asset. In the CBML model, the framework of spread volatility is similar to the mathematical modelling for the expected returns' volatility. Based on general foundations of asset pricing and simple computation, the CBML model can be suitably considered for variety of research in the field of asset pricing.

The paper is structured as follows. Section 2 describes the theoretical background of the prior research. A description of the dataset and distinct low-frequency spread measures is given in Section 3. Section 4 discusses the research findings, and these findings are concluded in Section 5.

2 Review of the Literature

In the literature of market microstructure, the financial market liquidity is one of the important disciplines. The microstructure of the financial market is concerned with details of how financial securities are executed at the time of trade. In the financial market, investors are possibly interested to anticipate costs associated with trading and effects of these costs on assets' prices. Liquidity influences market efficiency, trading cost, returns, and systemic financial stability (Chordia, Roll, y Subrahmanyam, 2001, 2008). Market liquidity is a multidimensional concept and it is described in distinct context. Lybek, Sarr, y and (2002) argue that a liquid market reflects various features: (a) low trading cost; (b) immediacy of transaction execution; (c) depth, in other words, the existence of limit orders; (d) breadth, which means small market impact of large orders; and (e) resiliency, indicates that new orders correct market imbalances.

Whilst liquid markets are described in distinct features, market liquidity can be defined in a number of ways. Market liquidity, in general, is the ease of trading an asset in the financial market. In other words, the immediacy of transaction execution with limited price impact and low transaction cost can be referred to higher liquidity. Market liquidity tends to be highly volatile in the financial market, which impose systemic liquidity risk (Guijarro, Moya-Clemente, y Saleemi, 2019). Liquidity is a time-varying risk factor, which interrelates with the transparency of information about assets' value (Bernales, Cañón, y Verousis, 2018), the number of liquidity providers and their access to capital (Brunnermeier y Pedersen, 2008), and an increased liquidity uncertainty which induces liquidity providers to ask for a higher compensation, that is, a higher executing cost (Ho y Stoll, 1981).

Distinct models, focused either on bid-ask spread proxies or volume-based liquidity measures, have been proposed to estimate the market liquidity in the financial market (Goyenko y cols., 2009). In general, the range of ask price for which a seller wants to sell an asset and bid price that a buyer wants to pay for an asset is referred to the liquidity and the cost of an asset, that is, the bid-ask spread (Cohen, Maier, Schwartz, y Whitcomb, 1981). A small size of spread is an indicator of higher liquidity. Moreover, ask and bid prices have been traditionally used as proxy of volatility (Garman y Klass, 1980; Parkinson, 1980). Starting with the Roll spread model, the literature in bid-ask spread has been gained tremendous development and various components, namely as, information asymmetry cost, immediacy cost, and order processing cost visualized in order to estimate the true spread.

Lesmond, Ogden, y Trzcinka (1999) introduced distinct spread proxies: (i) Zeros; (ii) LOT estimator of effective spread; and (iii) LOT Mixed estimator. Zeros estimator is the proportion of days with zero returns. The difference between percent buying cost and percent selling cost is referred to LOT estimator of effective spread. The LOT Mixed method estimates cost parameters based on maximizing the likelihood function of daily stock returns. Hasbrouck (2004) proposed a half-spread using Gibbs sampler Bayesian estimation of the Roll's model. Goyenko y cols. (2009) and Holden (2009) jointly proposed an Effective Tick estimator following the concept of price clustering, which is the probability weighted average of each effective spread size divided by average price. Corwin y Schultz (2012) introduced a high-low estimator of bid-ask spread, which is based on daily high and low prices. Fong, Holden, y Trzcinka (2017) proposed a measure of monthly spread proxy, which is a simplified version of the LOT Mixed estimator. Most recently, Abdi y Ranaldo (2017) constructed a spread model from close, high, and low prices (CHL), which is a modified version of Roll (1984).

3 Database and methodology

The scope of this study is to present a new estimation of the cost-based market liquidity, that is, the bid-ask spread. Additionally, this paper provides comprehensive comparison of the proposed strategy with distinct low-frequency spread proxies. The data used in this study contains daily observations of high, low, and closing prices, related to the S&P500 Index, and collected during the period 3 January 2001-30 October 2019. The analysis was executed on R programming software, where distinct econometric techniques, namely kernel density estimation (KDE) of the numerical distributions for the liquidity variables, the time-series analysis of the liquidity variables, and the correlation analysis, were applied. This section also looks at theoretical assumptions behind the construction of each applied spread proxy.

3.1 Roll Bid-Ask spread model

Roll (1984) assumes, that the true value of asset is based on random walk and independent of the order flow. Therefore, buy and sells orders are considered equally likely and serially independent. Under assumptions, that market makers bear only order-processing cost, this estimator is based on the serial covariance of change in prices.

$$RS_t = 2\sqrt{-Cov(\triangle P_t, \triangle P_{t-1})} \tag{6}$$

The shortcoming of the Roll bid-ask spread model is that the covariance of price changes can be positive. Therefore, the function of square root cannot compute spreads. Goyenko y cols. (2009) set a default numerical value to zero, when the sample serial covariance is positive. In this study, the Roll model is analysed as:

$$2\sqrt{-\frac{Cov(\triangle P_t, \triangle P_{t-1}}{P_t}}), \text{when } Cov(\triangle P_t, \triangle P_{t-1}) < 0 \quad (7)$$

$$0, \text{when } Cov(\triangle P_t, \triangle P_{t-1}) \geq 0$$

3.2 CS Bid-Ask spread model

Corwin y Schultz (2012) introduced a high-low estimator of spread. It is based on assumptions that daily high prices, H_t, are always buyer-initiated trades and daily low prices, L_t, are always seller-initiated trades. The CS model reflects market liquidity, trading cost, and volatility for two-consecutive single days.

$$CS_t = \frac{2e_t^{\alpha} - 1}{e_t^{\alpha} + 1} \quad (8)$$

where:

$$\alpha_t = \left(1 + \sqrt{2}\right)\left(\sqrt{\beta_t} - \sqrt{\gamma_t}\right) \quad (9)$$

$$\beta_t = \left[ln\frac{H_t}{L_t}\right]^2 + \left[ln\frac{H_{t+1}}{L_{t+1}}\right]^2 \quad (10)$$

$$\gamma_t = \left[ln\frac{max(H_t, H_{t+1})}{min(L_t, L_{t+1})}\right]^2 \quad (11)$$

The CS spread model further addresses overnight trading which may cause to produce negative spreads, where γ_t is larger in value than β_t. The negative bid-ask spread is a drawback of the CS spread measure. Whilst a spread is defined as ask-price minus bid-price, a spread is assumed a positive number in the literature of asset pricing. In order to deal with negative spread, researchers are suggested for various adjustments: (a) assume negative monthly estimates to zero; (b) set negative two-day estimates to zero and then compute the average; or (c) compute the spread for only positive estimates and take the average.

3.3 CHL Bid-Ask spread model

Most recently, Abdi y Ranaldo (2017) proposed a modified version of the Roll bid-ask spread model from daily close, high, and low prices (CHL). This model assumes, that the midrange of high and low prices on day t and day $t+1$ rely at the common close price of day t.

$$S_t = 2\sqrt{\left(ln(c_t) - ln\left(\frac{H_t + L_t}{2}\right)\right)\left(ln(c_t) - ln\left(\frac{H_{t+1} + L_{t+1}}{2}\right)\right)} \quad (12)$$

The CHL model is undefined, when the relationship between prices of day t and day $t+1$ around the common close price, c_t, causes of negative estimates. This implies, that the function of price variance fails to estimate spread for negative observed values. In such cases, Abdi y Ranaldo (2017) set default numerical values to zero.

$$S_t = 2\sqrt{max\left\{\left(ln(c_t) - ln\left(\frac{H_t + L_t}{2}\right)\right)\left(ln(c_t) - ln\left(\frac{H_{t+1} + L_{t+1}}{2}\right)\right), 0\right\}} \quad (13)$$

	N	Min	Median	Mean	Max	Std. Dev.	Skewness	Kurtosis
RS	4735	0.0000	0.0048	0.0081	0.1221	0.0112	3.35	23.01
CS	4735	-0.1098	0.0004	-0.0015	0.0468	0.0103	-1.66	13.09
CHL	4735	0.0000	0.0024	0.0032	0.0367	0.0037	2.77	16.51
CBML	4735	0.00000156	0.0048	0.0068	0.0766	0.0071	3.03	18.6

Table 1. The descriptive statistics of variables are computed from daily observations

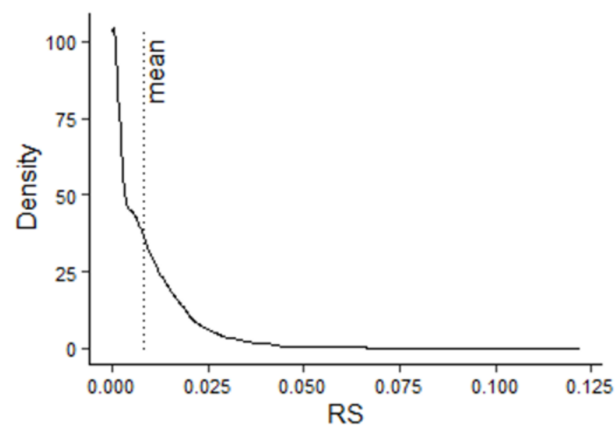

Figure 1. Density plot illustrating skewness for RS variable

The CHL spread is derived as:

$$CHL_t = \frac{1}{N}\sum_{t=1}^{N} S_t \tag{14}$$

Where N is the number of days in the month.

4 Results

The descriptive statistics of liquidity variables for the data sample are presented in Table 1, which vividly indicates the numerical differences among the spread proxies. As discussed earlier, each spread proxy is constructed under some specific conditions. The theoretical assumptions behind construction of each model would possibly impact the measurement of liquidity. As can be seen in Table 1, positive skewness indicates the right-skewed distributions of liquidity variables with values to the right of their mean. However, negative skewness is seen of the CS spread model, which indicates the left-skewed distributions of liquidity variable with values to the left of its mean. The higher kurtosis of liquidity variables is an indicator of extreme values in the dataset.

Figures 1–4 further provide an illustration of the numerical distributions for liquidity variables under the concept of kernel density estimation. Such non-parametric technique visualized the probability density function for each liquidity variable, while providing important quantity of information. Density plots clearly show differences in the numerical distributions of variables. The reason is, the Roll model and the CHL spread measure failed to compute around 32.65% and 21.37% observations, respectively. As mentioned earlier, the Roll model fails to estimate

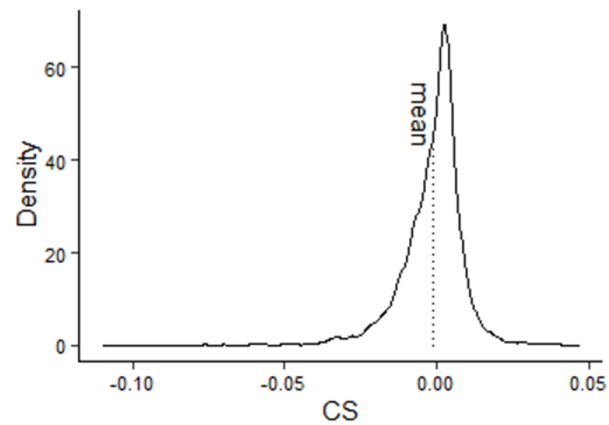

Figure 2. Density plot illustrating skewness for CS variable

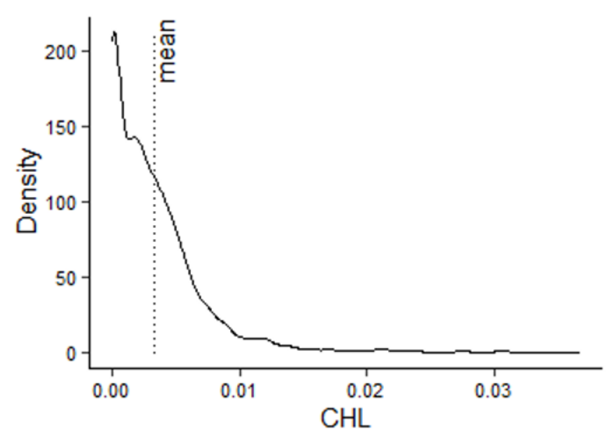

Figure 3. Density plot illustrating skewness for CHL variable

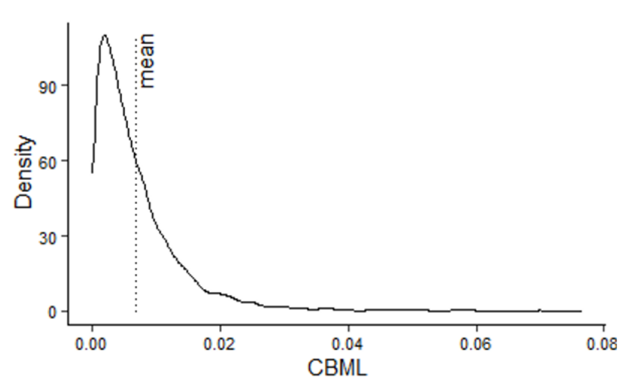

Figure 4. Density plot illustrating skewness for CBML variable

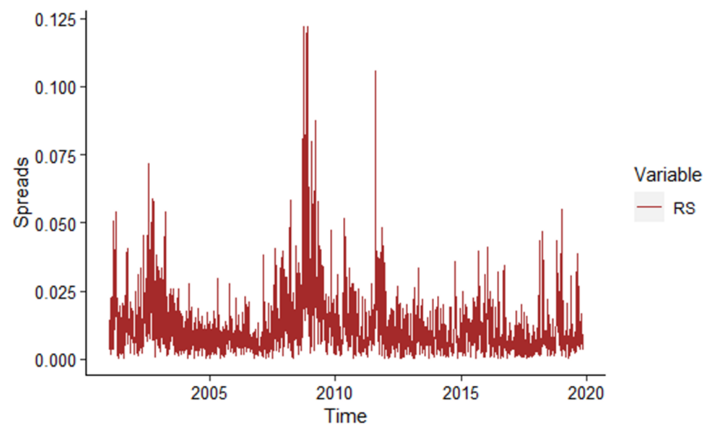

Figure 5. Time-varying cost-based market liquidity computed by RS variable

	RS	CS	CHL	CBML
RS	1	0.56	0.67	0.65
CS	0.56	1	0.67	0.45
CHL	0.67	0.67	1	0.46
CBML	0.65	0.45	0.46	1

Table 2. Correlation values among the spread proxies

spread when the covariance of price changes is positive, and the function of price variance in the CHL model cannot compute spread for negative estimates. In such cases, the study assumed all default numerical values to zero.

However, the CS model computed around 47.98% negative spreads in the data sample, which is clearly a violation of reality. As discussed earlier, a spread is always considered a positive number in the literature of asset pricing. The CS model was seen to produce negative spreads, when the maximum range of high-to-low price ratio for two-day period is larger than the expectation sum of price ranges over two consecutive single days. Most importantly, the CBML measure contains skewed shape of numerical distributions, while consistently estimating the positive spreads for the entire dataset.

Figures 5–8 show the cost-based market liquidity, while excluding all default numerical values of the Roll model and the CHL spread estimator, and negative spreads estimated in the CS model. However, the CBML variable is representing the cost-based market liquidity for the entire dataset. Despite shortcomings in Roll, CHL, and CS models, it was observed that the market liquidity is a time-varying risk factor and can suddenly disappear, as seen during the recent global financial crisis. Although, some noise in the liquidity has been occurring over time, but it is not persistent.

Table 2 shows the correlation coefficients among the spread proxies. In order to execute the analysis, the study considered only positive estimates in the CS model, and those observations for which the Roll and CHL models compute spreads. The results importantly revealed, that the CBML model has statistically strong correlation with the model proposed by Roll (1984), but the relationship of the CBML model is seen statistically moderate with bid-ask spread models proposed by Corwin y Schultz (2012) and Abdi y Ranaldo (2017).

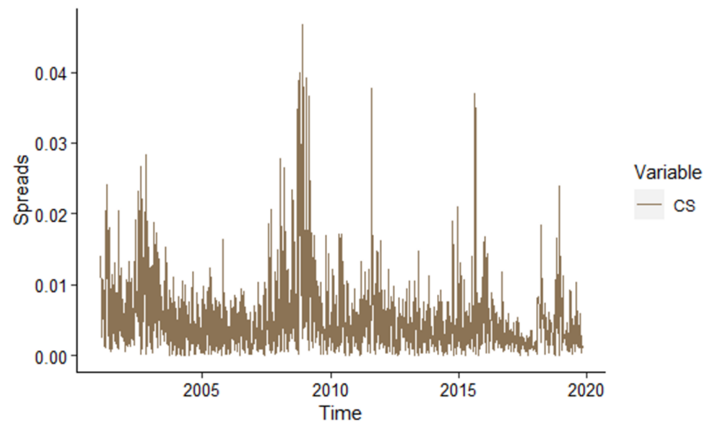

Figure 6. Time-varying cost-based market liquidity computed by CS variable

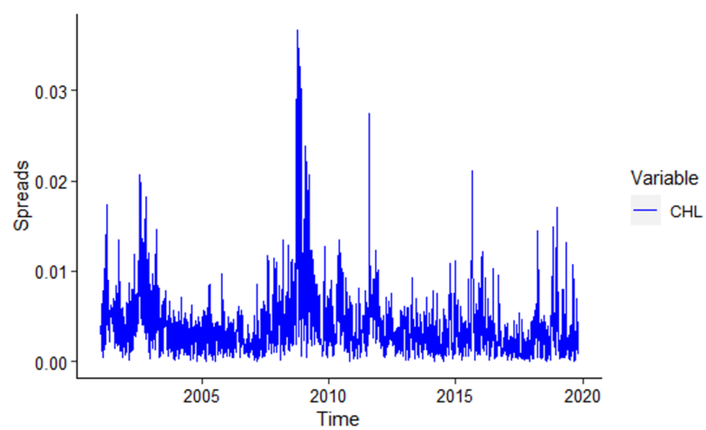

Figure 7. Time-varying cost-based market liquidity computed by CHL variable

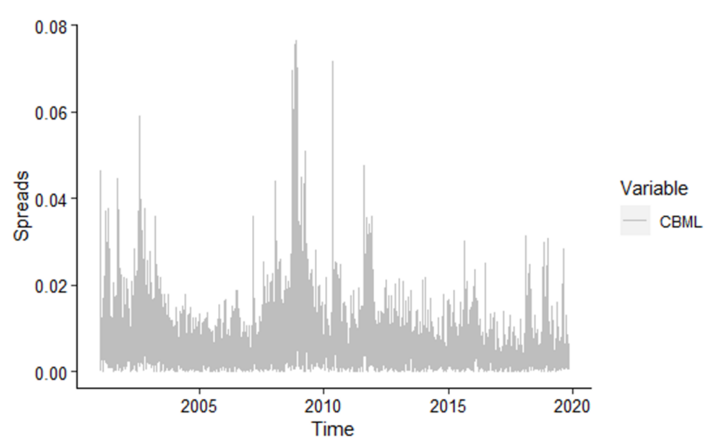

Figure 8. Time-varying cost-based market liquidity computed by CBML variable

5 Conclusions

This work constructs a new proxy of the cost-based market liquidity from daily high, low, and close prices. Compared with Roll and CHL spread proxies, the proposed method, CBML, consistently estimated the bid-ask spreads for an entire dataset, and utilized a wider set of daily information, namely high, low, and close prices. Unlike in the CS spread measure, the CBML measure steadily computed positive spreads. Additionally, the CBML model encounters the possible presence of an informed trading. Despite differences, the CBML proxy positively correlates with the applied spread proxies for the dataset, excluding the negative spreads of the CS model and the default values of the Roll and CHL models.

This estimation method for the cost-based market liquidity is straightforward, computationally less-intensive, and based on general foundations of asset pricing studies. Therefore, the proposed approach is suitable for variety of research. This research encourages researchers to study the proposed CBML proxy with a larger sample of liquidity measures, including the high-frequency spread measures. The future research would undoubtedly explore the significance of the CBML model in the study of asset pricing, corporate financing, and risk portfolio management.

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